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# *Effect of Imperfection on the Dynamic Buckling Composite Conical Shells Under Axial Impact*

K. T. Vipin, Priyadarsini R. S.
Dept. of Civil Engineering,
CET Trivandrum,
Kerala, India
ktvipin@gmail.com, priyadarsini_rs@yahoo.com

*Abstract* – **Laminated composite conical shells are extensively used in aerospace industry. Being thinner in section, these are susceptible to buckling when subjected to static or dynamic loading. The growing demand in safety of transport vehicles has also had a strong impact on the increasing interest in dynamic buckling. Thin walled shells prone to buckling are sensitive to imperfections which may reduce the buckling loads drastically. This paper presents the results of a numerical study on the behaviour of laminated composite conical shell with and without imperfection under dynamic axial impact. Composite conical shells of layup [0°/0°/60°/-60°]s and [0°/45°/-45°/0°]s were chosen for the current study and its behaviour under different impact durations were also studied. The magnitudes of imperfection in the shells were varied to study their effect on the buckling behaviour under impact loading. A comparative study of the dynamic buckling behaviour with their static one was also carried out.**

***Keywords — conical shell; composites; dynamic; buckling; axial impact***

## I. INTRODUCTION

A shell is a type of structural element which is characterised by its geometry, being a three-dimensional solid whose thickness is very small when compared with other dimensions. Shell structures possess various properties like high in plane stiffness, space containment, high strength to weight ratio, load carrying capacity etc., hence are extensively used in the fields of aerospace engineering, ship technology. Composites are materials which are formed by the combination of two or more different materials. They are preferred over their isotropic counterparts due to superior qualities in all fields of engineering. Laminated composites are widely used in aerospace applications. They form the body of aircrafts, fairings, payload etc. These structures are susceptible to wide range of loads, both static and dynamic.

Shells being thinner in section, buckling is the major condition of instability. Buckling can be broadly classified into two, static buckling and dynamic buckling based on the nature of load. Dynamic buckling can be further classified into vibration buckling and pulse buckling. The former relates to the response of structure to oscillatory loads and the latter relates to the response to impulse loads. The state of pulse buckling is more often encountered in the aerospace structures like ground impact of aircraft while landing, landing of the payload at final destination etc.

Imperfection can be defined as geometric or load irregularities in the structure. Initial geometric imperfection is one of the main reason for the discrepancies between the classical theoretical predictions and experimental results in case of thin walled shells under buckling. The presence of imperfection can affect the buckling strength of the shell considerably. Hence, their consideration in the numerical simulation is essential. In the absence of any raw data for imperfection, Koiter's theory can be employed to assess the effect of imperfection as they yield lowest buckling loads, hence the worst imperfections [1].

Very few works have been reported on the effect of imperfections on the dynamic buckling behaviour of composite conical shells, even though many works are available on the buckling of imperfection sensitive cylindrical shells under dynamic loads. Hutchison and Budiansky [2] carried out the dynamic buckling study on isotropic cylindrical shells using closed form solution technique. They assumed that the shells possessed imperfections similar to their eigen mode. Schokker et al. [3] performed numerical studies on the dynamic instabilities of the unstiffened and stiffened composite cylindrical shells under hydrostatic pressure. Pulse loading was applied for isotropic, anisotropic and orthotropic shells and their response were investigated.

Sofiyev [4] investigated the buckling behaviour of an orthotropic composite truncated conical shell with continuously varying thickness, subjected to a uniform external pressure which is a power function of time. Galerkin's as well as Ritz method were employed to obtain the dynamic buckling load and static critical loads. Effect of variation of semi vertex angle, power in the thickness expression, power of time in the external pressure expression were studied and it was found that these factors have appreciable effects on buckling behaviour of the shell. Sofiyev [5] studied the buckling of functionally graded truncated conical shells under axial loading. Galerkin's, Runge-Kutta methods and Wolmir criterion were applied to determine the dynamic buckling load. Influence of certain parameters like material compositions, loading speed, as well as shell geometries were also studied. Ajdari et al. [6] researched on the buckling behaviour of composite truncated conical shells under external loading by theoretical and numerical methods. Ritz method was employed for finding the dynamic stability load. The critical static and dynamic buckling loads were found analytically. Results of analytical calculations were compared with numerical results and with other analytical results from the literature and found that the static buckling load formula has very good agreement with other theoretical and numerical results. It was found that buckling pressure has a linear relationship with semi vertex angle. As geometric ratio is increased, the values of buckling loads decrease rapidly at first then remains constant. Showkati et al. [7] conducted experimental investigation on the buckling behaviour of conical shells under weld-induced imperfections on SCC (Shallow Conical Cap) and DCC (Deep Conical Cap) steel specimens. Initial depression of shell was created through welding, loaded under hydrostatic pressure. The results of the tests conducted in this study revealed that large imperfections strengthen the structure. The stiffening effect created by the local imperfection was higher for SCC specimens than the DCC cones.

## II. PROBLEM STATEMENT

Laminated conical shell made up of CFRP and a total length of 300 mm, base radius of 150 mm was chosen for the present study. The shell consists of 8 plies of 0.125 mm thickness each and semi vertex angle of 15º. Two layups $[0°/0°/60°/-60°]_s$ and $[0°/45°/-45°/0°]_s$ were chosen for the current study, where 0°corresponds to the meridional direction of the shell and stacking is done from outside to inside. The material properties for the layup is given in Table I [8].

**TABLE I. MECHANICAL PROPERTIES OF CFRP PLY**

| Longitudinal Tensile Modulus $E_{11}$ (N/mm$^2$) | Transverse Tensile Modulus $E_{22}$ (N/mm$^2$) | In-plane Shear Modulus $G_{12}$ (N/mm$^2$) | Poisson's ratio ν | Density (kg/m$^3$) |
|---|---|---|---|---|
| 134780 | 9250 | 4800 | 0.286 | 1700 |

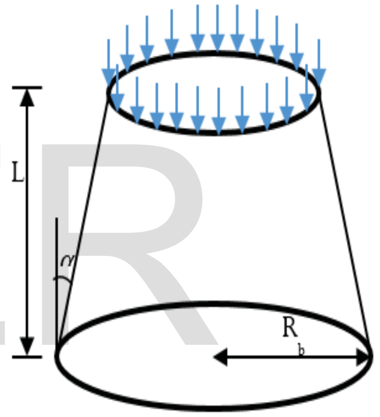
Imperfections were applied to the geometry using Koiter's theory. According to this, the linear buckling mode shapes are employed as the imperfection shape as it yields the lowest possible buckling load. The first static buckling mode extracted from eigen value analysis was applied as the imperfection shape with various amplitudes 'a' with respect to thickness of the shell 't' [9]. In the present study, imperfection magnitudes (a/t) ranging from 0, 0.05, 0.10, 0.25 and 0.50 were considered.

### A. Methodology

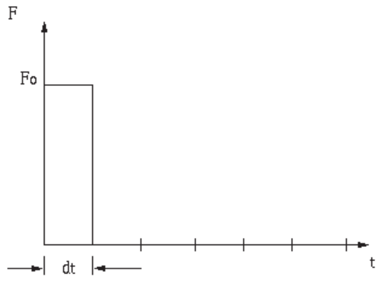
The buckling analyses was carried out using the finite element software ABAQUS. Initially, the buckling analysis of the shell subjected to axial compression was performed by linear static (eigen value) analysis and nonlinear static method using ABAQUS/Standard. Thereafter a frequency analysis is also done to determine the natural time period of the shell. The dynamic buckling analysis of shell under impulsive loading is carried out using ABAQUS/Explicit. For impact, rectangular loading is chosen, where a load of constant magnitude is applied suddenly for a finite duration. Different values of load duration, greater than and less than the natural period of the shell are considered for the study. The dynamic buckling loads are calculated using the Budiansky-Roth criterion. The dynamic buckling loads thus obtained are compared with static buckling loads.

### B. Finite Element Model

Composite conical shells are discretised using 4-noded S4R elements having six degrees of freedom per node, available in ABAQUS. S4R can be adopted for both static as well as dynamic analyses. Mesh convergence analysis was carried out and a model with 8580 elements, with 110 elements along the circumferential direction and 78 elements along the axial direction was chosen for analysis. At the base all the six degrees of freedom were arrested and at the loaded end only axial translation was allowed. Load was applied as uniformly distributed shell edge load and the loading diagram is as shown in Fig. 1(a).



(a) Conical composite shell with axial compression



(b) Step pulse load for loading shells

Fig. 1 Model geometry and loading diagram for the composite shells

## III. STATIC BUCKLING

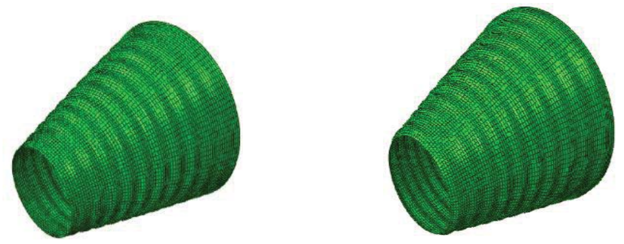
Under static buckling, two analyses namely linear static and nonlinear static analyses were carried out. Linear buckling analysis gives the theoretical buckling load for the perfect shells using linear shell theories. Always the linear buckling strength will be greater than the actual buckling strength since it does not take into account the nonlinear behaviour of the shell. Hence, a non-linear analysis using modified Riks method was also carried out to determine the actual load carrying capacity of the shell. Nonlinear analyses were conducted on shells by considering imperfections with magnitude as a/t = 0, 0.05, 0.10, 0.25 and 0.50. The results of the buckling analyses for the shells with layups $[0°/0°/60°/-60°]_s$ and $[0°/45°/-45°/0°]_s$ are reported in Table II. The buckling modes of the composite conical shells $[0°/0°/60°/-60°]_s$ and $[0°/45°/-45°/0°]_s$ obtained from the linear static analyses without imperfections are presented in Fig. 2.

TABLE II. STATIC BUCKLING LOAD (IN KN) FOR CFRP SHELL

| **Layup** | **Linear static buckling load (kN)** | **Imperfection Amplitude (a/t)** | **Nonlinear static buckling load (kN)** |
|---|---|---|---|
| $[0°/0°/60°/-60°]_S$ | 137 | 0 | 135 |
| | | 0.05 | 113 |
| | | 0.10 | 110 |
| | | 0.25 | 84 |
| | | 0.50 | 72 |
| $[0°/45°/-45°/0°]_S$ | 167 | 0 | 149 |
| | | 0.05 | 120 |
| | | 0.10 | 111 |
| | | 0.25 | 95 |
| | | 0.50 | 74 |

## IV. DYNAMIC BUCKLING UNDER AXIAL IMPACT LOADS

Dynamic Buckling of the shell subjected to axial impact load is investigated here. Budiansky-Roth criterion was followed in order to determine the dynamic buckling load. The load value, at which there exists a sudden change in response, is taken as the dynamic buckling load for that particular duration. The response at any finite point is monitored using displacement at that particular point for small values of loading parameter. For smaller loads, small oscillations were observed at loads lower than the actual dynamic buckling load. When the load reaches its critical value, the displacement-time history curve experiences a sudden jump. The lowest load at which a sudden change in response occurs is taken as the dynamic buckling load for that particular duration.


(i) Linear static (ii) nonlinear static

(a) $[0°/0°/60°/-60°]_s$


(i) Linear static (ii) nonlinear static

(a) $[0°/0°/60°/-60°]_s$

Fig. 2 Buckling modes obtained through static buckling analyses of perfect shells

In this study, axial impact in the form of step pulse is applied as uniformly distributed shell edge load having constant magnitude applied with finite duration. The loading diagram is as shown in Fig. 1(b). From the frequency analysis, it was found that the natural frequency of composite conical shell was found to be around 718 Hz and hence the natural period of the system around 1.4 ms. Based on this, five time durations of pulse loading were adopted namely, T = 0.5ms, 1ms, 1.5ms, 2ms and 5ms. Such a selection of load duration enables to understand the behaviour of shell under impulsive axial loading for different durations greater than and less than the natural period of the shell.

In order to get good results from the dynamic studies, the points for which displacements is to be monitored must be carefully chosen. Therefore, axial displacement at a unique point on the loaded end of the shell was considered i.e, for all the layups a particular point (node) at the loading end of the shell was considered. Fig. 3 presents the axial displacement of a point on loaded end of the shell with layup $[0°/0°/60°/-60°]_s$ without imperfections subjected to suddenly applied axial compression for a duration of 5ms for different loading magnitudes. Here, the curve corresponding to 85 kN show regular response and the cylinder vibrates about its equilibrium position. From 87 kN to 89 kN, there is a sudden change between two responses. This indicates the dynamic buckling condition and the load average equal to 88 kN is regarded as the dynamic buckling load of the shell $[0°/0°/60°/-60°]_s$ for that particular time duration. This criterion was used to estimate the dynamic buckling loads, analyses were performed for different time durations and the results are reported in Table III.

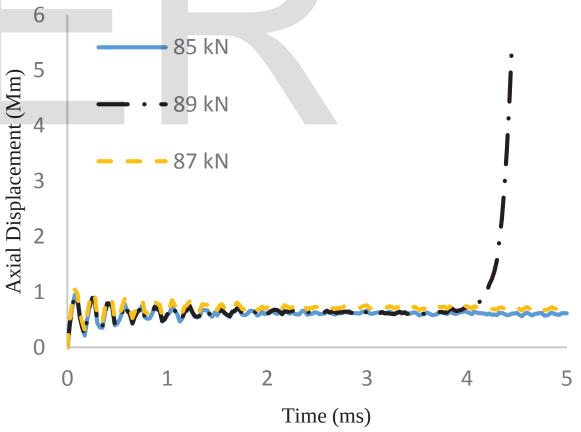
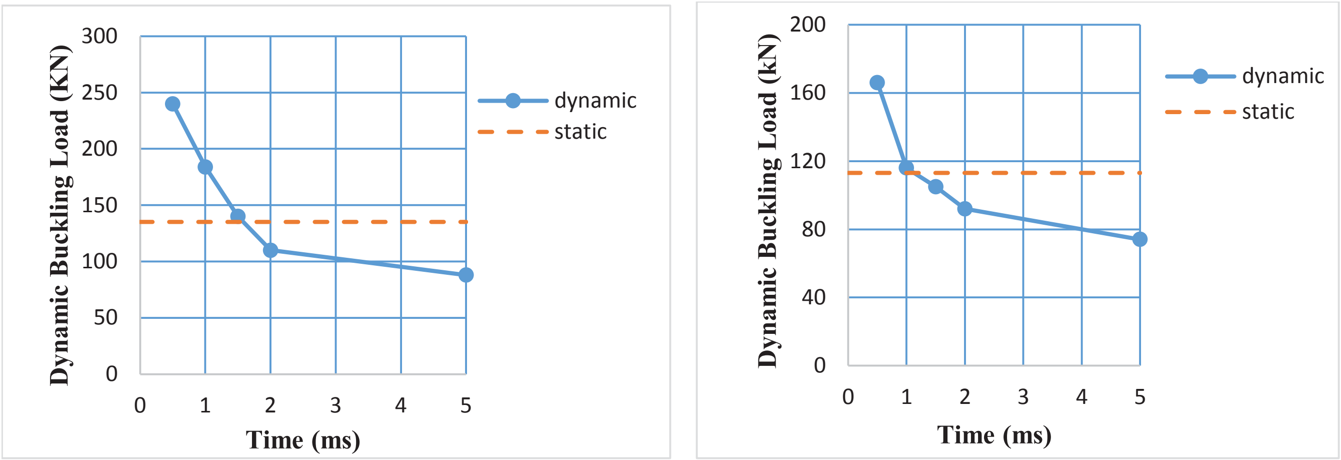
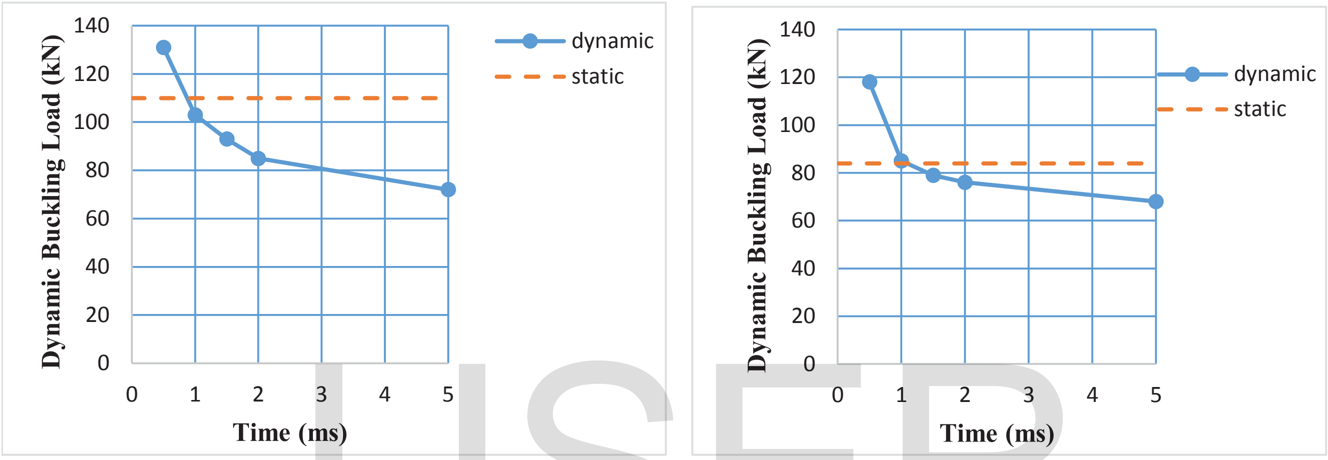


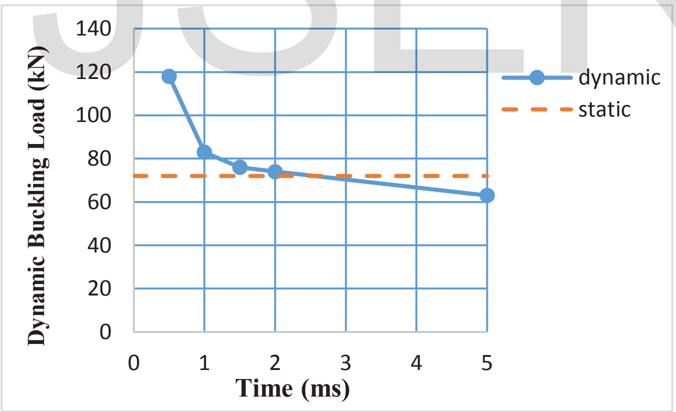
Fig. 3 Axial displacement for different load magnitudes on perfect composite conical shell $[0°/0°/60°/-60°]_s$



(i) a/t = 0 (i) a/t = 0.05



(i) a/t = 0.10 (i) a/t = 0.25



(v) a/t = 0.50

Fig. 4 Dynamic buckling loads for composite conical shell - [0°/0°/60°/-60°]$_s$ with different imperfection amplitudes

## V. RESULTS AND DISCUSSIONS

Dynamic buckling of composite conical shells due to axial impact was studied for the shells with layup sequences [0°/0°/60°/-60°]$_s$ and [0°/45°/-45°/0°]$_s$. Using static buckling analyses, the static buckling loads were found out for both the shells with and without imperfections. The dynamic buckling loads of composite conical shells with various imperfection amplitudes and for different load durations are plotted in Fig. 4. Each plot gives the comparison between the dynamic buckling loads with their corresponding static buckling loads. The dynamic buckling loads of composite conical shells were found to decrease with increase in load duration for all the amplitudes of imperfection. It was observed that as the imperfection amplitude was increased, the dynamic buckling load showed a declining trend for particular load duration. Also, the rate of decrease of load was found to be higher for composite shells having smaller magnitudes of imperfection. This trend was also observed in all shells irrespective of the layup sequence. In general, the dynamic buckling loads of the composite shells were found to decrease with increase in loading duration for all the imperfection amplitudes. The reason behind such a trend is the stress wave vibration that occurs between impacted and fixed end of the composite shells [9]. For duration less than the natural time period, the dynamic loads were found to be greater than corresponding static buckling loads. The dynamic buckling loads of the composite conical shells were found to vary under increasing imperfection amplitudes. The amplitude of the imperfections in the composite conical shells was found to influence their dynamic buckling loads more when the impact duration was smaller than the natural period. Hence the effect of amplitude on imperfections is significant in the case of smaller load durations and negligible in the case of longer impact durations. As the duration was found to increase, the dynamic buckling loads dropped below their static buckling loads.

TABLE III. DYNAMIC BUCKLING LOADS (IN KN) OF CFRP SHELL

| **Layup** | **Imperfection Amplitude (a/t)** / **T (ms)** | **0** | **0.05** | **0.10** | **0.25** | **0.50** |
|---|---|---|---|---|---|---|
| [0°/0°/60°/-60°]$_s$ | 0.5 | 240 | 166 | 131 | 118 | 118 |
| | 1 | 184 | 116 | 103 | 85 | 83 |
| | 1.5 | 140 | 105 | 93 | 79 | 76 |
| | 2 | 110 | 92 | 85 | 76 | 74 |
| | 5 | 88 | 74 | 72 | 68 | 63 |
| [0°/45°/-45°/0°]$_s$ | 0.5 | 166 | 144 | 131 | 123 | 123 |
| | 1 | 142 | 103 | 94 | 90 | 88 |
| | 1.5 | 127 | 92 | 90 | 88 | 88 |
| | 2 | 107 | 92 | 90 | 88 | 88 |
| | 5 | 87 | 87 | 83 | 76 | 70 |

From the Fig. 4 it is quite clear that the rate of decrease of load is high for smaller magnitudes of imperfection. Hence, the behaviour of composite conical shell with imperfection can be evaluated by considering small imperfection amplitudes as they lead to high percentage reduction in buckling loads. It is quite clear from this analysis that under load duration exceeding natural period of shells, using static buckling load as design load might be misleading.

## VI. CONCLUSIONS

The behaviour of composite conical shells with layups [0°/0°/60°/-60°]$_s$ and [0°/45°/-45°/0°]$_s$ under axial impact were studied for different durations. Effect of imperfection on the dynamic buckling strength of the shells were also studied using Koiter's theory. The dynamic buckling loads were found to be larger than static load values when the time of application of loads were less than the natural period of the shell. When the load duration was increased, the dynamic buckling load values showed a declining trend and for duration greater that the natural period the dynamic buckling loads were found to be lower than their corresponding static buckling loads. Also, as the imperfection amplitude was increased, the buckling strength of shell was found to show a declining trend. The rate of decrease of load was found to be higher for smaller magnitudes of imperfection. In general, the present study proves that for design purpose taking the static buckling load might be unsafe under pulse loading for longer durations.

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